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具备“反射光学”之技：约翰·迪伊早期对混合数学的沉浸

“If You Were Skilled in ‘Catoptrics’”: John Dee’s Early Immersion in Mixed Mathematics

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摘　要：本文立足于迪伊在1550–1570年代的学术语境，考察其早期科学思想在近代早期学科复杂性背景下的形成机制。文章指出，迪伊发展出一种融合数学与物理学的混合数学方法，不仅是为了回应16世纪欧洲打破数学与自然哲学学科界限的思想潮流，同时也是其彰显经院哲学所谓“隐秘的质”的重要路径。研究表明，在迪伊新近建构的“占星物理学”体系中，若要精确计算并操控天体所具有的隐秘的质或影响，关键在于掌握几何光学方法，尤其是反射光学。迪伊光学方法的理论基础，一方面继承了中世纪光之形而上学传统，另一方面则吸收并融合了文艺复兴时期的自然魔法思想。

关键词：约翰·迪伊　混合数学　隐秘的质　光之形而上学　几何光学

Abstract: This paper aims to understand Dee’s early scientific ideas (1550s-70s) on their own terms and within the context of early modern disciplinary complexities. I argue that Dee developed his new mixed-mathematical approach to physics not only to engage with the sixteenth-century European movement to break the disciplinary barriers between mathematics and physics, but as his own way to manifest the scholastic “occult virtues”. The paper claims that for Dee the key to calculate and manipulate celestial hidden qualities or influences within his newly developed discipline of “astrological physics”, was to be adept in geometrical optical methods such as in “catoptrics”. The foundation of Dee’s optical approach is shown to be indebted to traditions of both Medieval light metaphysics and Renaissance natural magic.

Key Words: John Dee; Mixed mathematics; Occult virtues; Light metaphysics; Geometrical optics

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Introduction

As a polymathic and even legendary early modern astrologer who served Queen Elizabeth I at court, John Dee’s (1527-1608) life and works have received abundant scholarly studies which have borne fertile fruits in the last several decades. Scholarship on Dee has been focused on diverse topics to date, including his political position and religious interests; his travels throughout Europe and patronage; his ranging over all branches of learning, including studies in ancient history, astrology, mathematics, cryptography and theories of navigation; his library

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catalogue which was claimed to contain over 3,000 books and 1,000 manuscripts; his "notorious" private "spiritual diaries" of "angelic conversations", and his place in Renaissance intellectual history. These rich scholarly efforts have been fully presented and efficiently overviewed in Nicolas H. Clulee's recent chapter, "John Dee at 400: Still an Enigma" in the edited collection *Bridging Traditions* (2009). In this survey, Clulee clarifies and compares all different approaches to, and the contexts of, the so called "Dee industry" developed over the past century. Clulee not only differentiates the terrain of Dee scholarship between historical and more cultural studies, but also subdivides the former between those attempting to free Dee completely from scientific revolution studies (or even from any historiographic theme in the history of science), and those tending to place Dee's science or natural philosophy as central to his own ideas and locating him firmly in the Scientific Revolution. In line with the attempts to uphold and centralise Dee's science, pioneered by Francis R. Johnson and Eva G. R. Taylor, and continued through the Warburg school, Clulee concludes by suggesting that Dee "remains an enigma" whose works still elude complete understanding, especially with regard to "current developments in natural knowledge, mathematics, and science." [1-3]

Attempts to understand Dee's scientific ideas on their own terms and in their most suitable contexts associated with the Scientific Revolution, to "rescue" Dee from obscurity, as Clulee claimed, indeed have blossomed in the past four or five decades. Dee was studied as a significant example in the relation of the occult sciences (such as astrology and alchemy) to the Scientific Revolution, and was thus located within the historiographic theme of "science and magic"—both of which topics increasingly became a major concern for early modern historians of science from the 1960s, and continues to thrive in current scholarship. The complex Renaissance intellectual contexts of Dee's magic, for example, has been shown to be indebted to specifically medieval light metaphysical traditions going back not only to the work of Robert Grosseteste and Roger Bacon, but to Ficino and even to AI-Kindi. Dee's optical theories of celestial influence in the *Propaedeumata aphoristica* (1558) were also understood as including a spiritual dimension so that the scientific arguments pertaining to natural magic could be reconciled with his later "angelic conversations" influenced by other branches of magic–a "one mind" or "single minded" Dee with coherent thinking, as Clulee and others called it ①.

This article partially adopts these approaches in the above direction to position Dee's science within these Renaissance intellectual contexts and to show his debt to optical theories and the traditions of light metaphysics and natural magic. However, the article aims to shed some new light on Dee's key role in the so-called Scientific revolution rather than as merely a figure who straddled science and magic. Firstly, it will pitch into the topic through the perspective of the disciplinary complexities during the early modern period, in particular, the disciplinary realignment between mathematics and natural philosophy. Historians of early modern science have largely considered the crisis of the Scholastic philosophy and the birth of modern science as resulting from important disciplinary transformations during the sixteenth (and early seventeenth) centuries. This article will set out to strengthen Dee's relation to the scientific revolution more directly within this context, therefore, by focusing on Dee's new mathematical physical method developed both in his the *Propaedeumata aphoristica* and in the renowned "Mathematicall Praeface" that he added to Henry Billingsley's translation of Euclid's *Elements* (1570). Secondly, the gradual emergence of new disciplines during this time, as historians have claimed, can also be seen as being revealed in the light of the reconsiderations and fragmentations of some important scholastic notions. The focus of this

① For the scholarly trajectory that situates Dee within the context of the Scientific Revolution and argues for the coherence of his thought, see: Allen G. Debus, *The English Paracelsians* (London: Oldbourne, 1965); Francis R. Johnson, *Astronomical Thought in Renaissance England* (Baltimore: Johns Hopkins Press, 1937). For the "one mind" thesis and recent studies, see Stephen Clulee, *John Dee's Natural Philosophy: Between Science and Religion* (London: Routledge, 1988); Stephen Clucas, ed., *John Dee: Interdisciplinary Studies in English Renaissance Thought* (Dordrecht: Springer, 2006).

paper, accordingly, will be on the role played by the previously overlooked context of the disassembling of scholastic “occult qualities” in the process of these disciplinary changes. And Dee should be considered along with other innovative early modern natural philosophers who were handling this scholastic notion by developing new approaches to physics.

I begin this paper by looking at the backgrounds of both the traditional disciplinary divisions between mathematics and natural philosophy, and the Renaissance non-scholastic and natural magical notion of celestial occult virtues represented by Cornelius Agrippa, who proposed that all hidden qualities in nature should be explored by all possible means—philosophical, magical, and mathematical. Against these backgrounds, I will then show that the Elizabethan magus Dee, having gained belief about the close relevance of mathematics to natural philosophy during his time at Louvain, developed a version of geometrical optics derived from the Middle Ages light metaphysics tradition, both in his the *Propaedeumata aphoristica* and in the “Mathematicall Praeface” when he was in his early career located at Cambridge University. Based on this mixed mathematical method, Dee advanced a new discipline of “astrological physics” to break the traditional disciplinary barriers between mathematics and natural philosophy. The key to calculate and manipulate certain celestial hidden qualities within the new discipline, as Dee claimed, was to be adept in certain geometrical optical methods such as in “catoptrics”.

I. Disciplinary Divisions, Natural Magic, and Celestial Occult Virtues

There are two aspects of the background that need to be considered before looking at Dee’s works. The first context that requires some brief consideration concerns the disciplinary divisions between mathematics and natural philosophy up to the early modern period. The goal of natural philosophy, according to ancient scholastic natural philosophers, was to provide a “demonstratio propter quid” for any given natural phenomenon, i.e., a “demonstration according to which” which must refer to causes; but what mathematics could provide was what the scholastics called “demonstratio quia”, or “a demonstration that”, i.e., simply a demonstration of the fact, but not why or how this fact occurred. [4] In Aristotelian terms, furthermore, the importance of mathematics can at best be seen in the fields of applied mathematics (e.g., optics, astronomy, mechanics etc.), which were categorised as “intermediate sciences” (*scientiae mediae*). Whereas pure mathematics dealt with pure quantities (e.g., number or extension) without “sensible” qualities, the scientiae mediae handled the quantitative aspects of sensible qualities (e.g., weight, or the geometry of light rays). However, as these intermediate sciences were not conceived as helping to understand the causes of things, they were still thought of as only peripheral to natural philosophy. In general, these boundaries were preserved strictly through the Middle Ages and the early stage of the Renaissance period, and in the light of this classification, mathematicians, including astronomers, remained a separate group from natural philosophers until the sixteenth century when things began to change.

It was during the sixteenth century that new developments (e.g., the voyages of discovery, the Copernican theory etc.) began to challenge and undermine the assumptions of Aristotelian philosophy, and increasingly more accurate mathematical models came to be seen as the way to deal with phenomena in the real world, such as Copernican astronomy. Consequently, the traditional scholastic separation of physics and mathematics in the university curriculum began to encounter severe challenges. As a result, the alternative mixed mathematical sciences (as the intermediate sciences came to be called through the Medieval period) provided new models of scientific practice. Mathematics became increasingly implicated in the ways of understanding natural phenomena, and of generating true and universal knowledge of nature. Over more than a century, the issue as to whether mixed mathematics was subordinate to, or an essential part of natural philosophy, was urgently debated across Europe. As is generally acknowledged, this debate started firstly as a result of responses by mathematical practitioners across a range of mathematical sciences and mechanical arts–optics, astronomy, ballistics, cartography, navigation, architecture, and others—

to the old disciplinary separation. [5] Subsequently, as a number of sixteenth-and early seventeenth-centuries innovative natural philosophers, including Dee, recognised the importance of mathematics for understanding the workings of nature—workings which increasingly came to be seen in the seventeenth century, under this mathematical impulse, as mechanical in nature.

Another context that also needs to be taken into account concerns the Renaissance belief in natural magical notions of celestial occult virtues, for example, as represented by Cornelius Agrippa. During the Renaissance, innovative philosophers very often saw the magical world picture as providing a promising replacement for Aristotelian cosmology, and for them the disciplinary division of mathematics and physics was always bound up with natural magic, as a way of coming to grips with the issue of scholastic "occult qualities". This complex disciplinary intertwining in the sixteenth century can be seen, for example, in Cornelius Agrippa's first chapter of Book II of *Three Books of Occult Philosophy* entitled "Of the necessity of Mathematicall Learning". Agrippa insisted that "the Doctrines of Mathematicks are so necessary to, and have such an affinity with Magick, that they that do profess it without them, are quite out of the way, and labour in vain, and shall in no wise obtain their desired effect." [6]

In an even more powerful statement, Agrippa told his readers that

> Whosoever is desirous therefore to study in this Faculty [natural magic], if he be not skilled in naturall Philosophy, wherein are discovered the qualities of things, and in which are found the occult qualities of every Being, and if he be not skillful in the Mathematicks, and in the Aspects, and Figures of the Stars, upon which depends the sublime virtue, and property of every thing… he cannot be possibly able to understand the rationality of Magick. [7]

Agrippa considered both natural philosophy and mathematics as essential components of natural magic ① Handling or understanding "occult qualities of every Being" in nature so that they can be included within true physics was the central aim of all groups of mathematicians, natural philosophers and natural magicians. Agrippa himself also provided a general understanding of the "occult qualities" during the Renaissance:

> There are...vertues in things, which are not from any Element, as to expell poyson, to drive away the noxious vapours of Minerals, to attract Iron, or anything else…whence also it being litle in quantity, is of great efficacy; which is not granted to any Elementary quality…And they are called occult qualities, because their Causes lie hid [from our sense], and mans intellect cannot in any way reach, and find them out. (6, chapter XII, p.167)

According to the Ancient and Medieval Scholastic terms, manifest qualities were those sublunar properties or virtues whose causes derived from Aristotelian primary dichotomies of hot, cold, wet, and dry; and sensible qualities such as hardness or softness can be reducible from them. It was maintained that because they can be sensed directly, they give rise to true scientific knowledge; natural philosophy had to be based upon sensible experience that was familiar to everyone (and therefore could not give rise to disagreement). In this process, it was assumed, human sensory organs firstly received the forms of manifest qualities (abstracted from their subject matter), and then imagination further abstracted the sensible forms until finally human intellect examined all of them, separating the accidentals from the essentials until it obtained the universal forms. The "occult qualities", on the contrary, were characterized as those whose causes were other than those of the manifest causes, and the sensible qualities derived from them; so they were insensible, and hence could not be grasped by the human soul, and thus were also unintelligible. As such they were excluded from true natural philosophy

① "There is no work that is done by meer Magick", Agrippa wrote, because magic also incorporates the learning of what he calls the three faculties: natural philosophy, mathematics, and theology.

according to Aristotelian definitions—an insensible quality that was not a familiar part of everyone's daily experience could not form part of natural philosophy that was defined in terms of familiar sensible experience. This is exactly what Agrippa was referring to above when he says "There are… vertues in things, which are not from any Element"—manifest properties are located in one or other of the four Elements, but there are some properties which are not derived from any of the Elements or their sensible qualities.

For Agrippa, therefore, "occult qualities" were those qualities, virtues, or properties whose "Causes lie hid". Through natural magicians' eyes, even though those hidden causes of phenomena could not be sensed and reached directly, nevertheless they reveal inner and deeper "secrets" of nature. Exploring these qualities in a proper way, as Agrippa had suggested, by adopting proper mathematical, philosophical and magical methods, was thus a tough challenge for natural magicians. Especially since Agrippa, just like many other philosophers in this time, did not just address this concept mainly within the sublunary region, but also with regard to the celestial realm. As Agrippa stated above, understanding the "occult qualities of every Being" in nature included "the Aspects, and Figures of the Stars, upon which depends the sublime virtue, and property of every thing". In fact, this reflected a notion shared by scholars during this time especially by astrologers and innovatory physicians, that there were occult heavenly virtues having impacts on terrestrial phenomena in such a way that we can only know their effects (e.g., generation and corruption), but not how they cause those effects. Those hidden causes in the heaven, for each astrologer could be imagined in different ways. To the French physician Jean Fernel, for example, the celestial realm was completely "divine"; and for him the meaning of "divine" was virtually "occult", so he considered celestial influence to be simply "occult". For Agrippa, it can refer to both terrestrial occult qualities and "the sublime virtue" in the heaven. The challenge Agrippa really set up here was to explore "everything" in both the celestial and terrestrial regions. Although "their Causes lie hid", he believed one way to understand them was through the mathematical approach–hence, he wrote "Of the necessity of Mathematicall Learning".

Against these backgrounds, it is not surprising that mathematical teaching and practice in the English universities, such as at Cambridge where Dee was based until the 1570s, frequently combined the Aristotelian curriculum with private magical interests and tutorials. Pursuing such interests was not uncommon in the late sixteenth and early seventeenth centuries. The works of such Renaissance magi as Marsilio Ficino, Agrippa, and Giordano Bruno were quoted very often as authorities in notebooks and other writings of both tutors and students within the universities. In the rest of this article, we shall consider John Dee, originally from Cambridge, as a typical example of this larger phenomenon. [8]

II. John Dee Breaking the Disciplinary Barriers

Dee, matriculated at St. John's College in 1542, and was then appointed at Trinity College, Cambridge, in 1546. He soon left to study at the University of Louvain in the Southern Netherlands, where he made contact, and in some cases close associations, with a number of leading mathematical practitioners, including Gemma Frisius, Gerardus Mercator, Gaspar à Mirica, Antonio Gogava, and Pedro Nunes. [9] He returned to England in 1551 and continued promoting mathematical studies, mostly by teaching back at Cambridge. In 1558 Dee published his first significant work, *the Propaedeumata aphoristica*, to explore "Certain Outstanding Virtues of Nature". [10] by committing to the mixed-mathematical method. But it is now known that Dee developed these ideas earlier, in response to an attack on the work of the leading mathematician of the day (and a personal friend of Dee's), Pedro Nunes, by Diogo de Sá in his *De navigatione*. [11] Dee's copy of De Sá's book, which he acquired in 1552, is heavily annotated, and the notes reveal Dee's belief not only in what a recent commentator has called "the mathematical principles of nautical science", but also in "the power of mathematics to construct valid knowledge."(9, p.463; 465) De Sá takes the traditional view that mathematical knowledge is limited and superficial

when judged from a philosophical point of view and is vigorous in his opposition to what has been called Nunes's "program for the mathematization of the real world." (9, p.462) From Dee's annotations to De Sá's criticisms of Nunes, Dee was already committed to the view that mathematics could make a much more valuable contribution to natural philosophy than scholastics allowed. The importance of this for Dee was subsequently made public in his *Propaedeumata*. In this work, Dee announces his intention in the dedication to the famous cartographer, Gerard Mercator, with whom he worked at Louvain. Addressing the mathematician and instrument-maker Mercator as "renowned mathematician and philosopher", Dee writes of them philosophizing together. Dee's book, subtitled "on certain more efficacious virtues of nature", not only proceeds demonstratively, but also establishes "the main principles of the science." It is therefore rightly called by Mercator, Dee's "great demonstrative work". (10, p.111; 173)

Dee's approach in this work is essentially to use the principles of geometrical optics to establish the way all virtues (including occult ones) are propagated in nature (all virtues are assumed to be propagated in the same way that rays of light are), and this then enables him to develop a mathematical theory of astral influences (that he believed met Aristotle's criteria for a demonstrative science precisely) because the rays of virtue which Dee saw as conforming to the principles of geometrical optics were the fundamental causal mechanisms operating throughout the whole cosmos. [12] He continued to point to the importance of mathematics in the renowned "Mathematicall Praeface" that he added to Henry Billingsley's translation of Euclid's *Elements* (1570). This was supposedly addressed to "unlatined people and not Universitie Scholars", and because these readers might not have been aware of disciplinary disputes about the relationship between mathematics and natural philosophy, Dee made his claims about the mathematical principles of natural philosophy extremely clear throughout. [13] Even in his opening paragraph we read that both Platonic idealists and more down to earth Aristotelians will benefit from what Dee has to tell them about mathematics: "the *Pythagoricall*, and *Platonicall* perfect scholer, and the constant profound Philosopher, with more ease and spede, may (like the Bee,) gather, hereby, both wax and hony." ① Dee goes on to claim that mathematical entities exist in between supernatural beings and natural things: "not so absolute and excellent, as thinges supernatural: Nor yet so base and grosse, as things naturall". ② Shortly after he draws support from "the great & godly philosopher" Boethius, and "the noble Earle of Mirandula", Giovanni Pico, Boethius is quoted as saying that "All thinges (which from the very first originall being of thinges, haue bene framed and made) do appeare to be Formed by the reason of Numbers. For this was the principall example or patterne in the minde of the Creator." Similarly, Pico is quoted as saying "By Numbers, a way is had, to the searchyng out, and vnderstandyng of euery thyng, hable to be knowen." ③ According to Dee, the "profound and diuine Science" of mathematics will take the "zelous Philosopher" beyond mere contemplation to certainty and truth④.

In short, Dee's "Mathematicall Praeface" presents the study of mathematics as far more important than was ever allowed in the scholastic tradition. His belief that mathematics should be an indispensable aspect of natural philosophy could hardly be missed. As he wrote at one point: "O comfortable allurement, O rauishing perswasion, to deale with a Science, whose Subiect, is so Auncient, so pure, so excellent, so

① Dee, *The Mathematicall Praeface to the Elements of Geometry of Euclid of Megara (1570)*, ed. Debus (1975). Quotation from sig. *iiir*.

② Ibid.

③ Ibid., sig. *ir-v*. Dee is quoting from Boethius, *De institutione arithmetica*, I, 2; and from the eleventh of the "Conclusions Mathematicall" from Giovanni Pico della Mirandola, *Conclusiones nongentae in omni genere scientiarum* (Rome, 1486). For modern editions, see Boethius, *Boethian Number Theory: A Translation of the De Institutione Arithmetica*, trans. Masi (1983); Giovanni Pico della Mirandola, *Syncretism in the West: Pico's 900 Theses (1486): The Evolution of Traditional, Religious, and Philosophical Systems*, trans. Farmer (1998).

④ Dee, *Mathematicall Praeface, sig. iv.*

surmounting all creatures, so vsed of the Almighty and incomprehensible wisdome of the Creator, in the distinct creation of all creatures: in all their distinct partes, properties, natures, and vertues.” (Dee, Mathematicall Praeface, sig. ir.).

Furthermore, Dee’s notion that mathematics could reveal truths about the natural world derived, at least in part, from his commitment to natural magic, in line with what we saw in Agrippa’s claim. For example, Dee’s list of the books he had acquired between 1557 and 1559, shows that he obtained Agrippa’s *De occulta philosophia* at the same time that he procured Pico’s 900 Conclusions and Marsilio Ficino’s influential magical treatise *De triplici vita*, and it was obviously a source from which he drew heavily. This is important for our purposes, because as we have seen in the last section, Agrippa not only insisted upon the link between mathematics and magic, but he also insisted that natural philosophy was an essential part of magic. Dee shows the same tendency to disregard the disciplinary distinctions between philosophy, mathematics, theology, and of course magic. This can be seen in the numerous places throughout the “Praeface” where Dee claims that mathematics not only reveals the arrangement and structure of things, but thereby indicates the causal relations between things—in making claims like this, Dee is rejecting the scholastic separation of mathematics from philosophy on the grounds that mathematics cannot lead us to causes. Consider, for example, his account of “Perspective”, where he writes:

> Againe, of thinges being in like swiftnes of mouing, to thinke the nerer, to moue faster: and the farder, much slower. Nay, of two thinges, wherof the one (incomparably) doth moue swifter then the other, to deme the slower to moue very swift, & the other to stand: what an error is this, of our eye? Of the Raynbow, both of his Colours, of the order of the colours, of the bignes of it, the place and heith of it, (&c) to know the causes demonstratiue, is it not pleasant, is it not necessary? of two or three Sonnes appearing: of Blasing Sterres: and such like thinges: by naturall causes, brought to passe, (and yet neuertheles, of farder matter, Significatiue) is it not commodious for man to know the very true cause, & occasion Naturall?

The science of perspective does not just provide a demonstration of what can be seen (demonstratio quia), but explains why these things are seen the way they are (demonstratio propter quid). Similarly, for Dee, it can explain why a peacock’s tail and a dove’s neck look the way they do, and why an oar in water seems to be broken “by demonstration Opticall, the order and cause therof, is certified: euen so, as the effect is consequent. Yea, thus much more, dare I take vpon me, toward the satisfying of the noble courrage, that longeth ardently for the wisedome of causes natural”.

Similarly, in his description of “Astrologie” as another example, Dee insists that it is based on “Not onely (by *Apotelesmes* [the influence of the stars]) τὸ ὁτὶ. but by Naturall and Mathematicall demonstration τὸ διότι.”(12, pp.61-64) These Greek expressions, meaning “the fact that” and “the reason that”, respectively, are the Greek equivalents of *demonstratio quia* and *demonstratio propter quid*. Accordingly, Dee is claiming that the hidden influences of the stars are not just stated as facts in astrology but are explained by means of natural and mathematical demonstrations.

It is clear, therefore, that Dee can be seen as an important promoter of the increasingly acceptable view that mathematics could, and did, provide knowledge of causes, and demonstrations propter quid, and should be seen, therefore, as making a valuable contribution to the early modern new mathematical approach to natural philosophy. Furthermore, Dee’s magical perspective also stimulated his attempts to establish the relevance of mathematics to natural philosophy. So far, we have been discussing Dee’s new mathematical approach to natural philosophy in general terms. There is more to be learned, however, from looking in detail at the way Dee developed his methodology. As we shall see, it developed out of his attempts to establish astrological influence upon a more certain foundation.

III. Geometrical Optics and Dee’s Mathematical Treatment of Celestial Influence

Apart from these general statements above,

Dee had a specific notion that mathematics can reveal the arrangement and structure of things, and hence indicate the causal relations between things, in particular, the way the occult virtues of things operated, and thereby could contribute to natural philosophical understanding. This notion originated in work that he did at Louvain to base astrology on a more secure foundation, in response to the damaging criticism of astrology by Giovanni Pico, in his *Disputationes adversus astrologiam divinatricem* (*Disputations against Divinatory Astrology*) ① . Dee was one of those with "contemporary concerns about astral powers" as a result of Pico's attack on astrology and the new Copernican theory which was, at least in part, a response to Pico. [14] And it was Copernicus's new theory that led Dee to think that it should be possible to calculate the different strengths of the astral virtues or powers on earth, and thereby improve understanding of astrological causes and their effects.

The important point about the Copernican theory from Dee's point of view was that it enabled astronomers to establish the distances of the Sun, moon, and planets from the earth. Assuming that influences from the heaven were propagated in the same way that light was emitted from the celestial bodies, the more distant a body was from the earth, the weaker would be its power. The Copernican theory enabled calculations to determine the distances of heavenly bodies from the earth, and therefore the relative strengths of their influences on the earth. Just as the celestial bodies emitted rays of light, so they emitted rays of whatever other occult virtues and influence they were supposed to have, according to astrological traditions. Given that a light source is less intense the further away it is from an observer, so a planet's influence on the earth would be less intense the further away it is. Similarly, a larger light source would have a greater effect than a smaller, although it too would diminish with distance. Dee also noted that the angle of incidence of the rays from a celestial body on the surface of the earth would also affect the power of those rays. Geometrical optics, therefore, provided the mathematical techniques for determining astrological influence, or the occult virtues from the heaven②.

This would have been unthinkable before the appearance of Copernican theory. In the geocentric system of Claudius Ptolemy it was impossible to determine even the order of the planets, much less their distances from the supposedly central earth. The order of the heavenly bodies in the Ptolemaic system was established only by convention, because geometrically, they could be in any order. For the outer planets, the time for a complete orbit of the sky was taken to indicate their order (Saturn, taking 30 years to circle the earth, was assumed to be the furthest away; Jupiter at 12 years was next, and Mars at two years was next). The order of the sun, Mercury and Venus, however, was more controversial. Some astronomers held Mercury and Venus to be above the sun, between the sun and Mars, while others held them to be below the sun, between the sun and the moon, which was the closest to the earth. However, it was acknowledged that even Saturn could be below the sun, and the appearances and the mathematics would remain the same. (4, pp.48-61)

Astrologers before Copernicus, therefore, had technical difficulty in thinking of connecting the powers of celestial influence with planetary distances. Astrological influence in the Ptolemaic tradition was considered to vary in accordance with so-called "Aspects", that is to say the angular formations the heavenly bodies made to one another against the backdrop of the fixed stars. Planets in conjunction —appearing close together in the sky—had a strong effect on each other and might reinforce one another or cancel one another out with regard to the earth. Planets in opposition – opposite to each other across the sky—had opposing effects on the earth. Planets separated by 90°, by 60°, or by 30° were held to have correspondingly different effects on one another and on the earth. The nature of causation, according to Aristotle's model of four causes (material, formal,

①Giovanni Pico della Mirandola, *Disputationes Adversus Astrologiam Divinatricem* (1496). For a major study of the impact of this work, see Westman, *The Copernican Question: Prognostication, Skepticism, and Celestial Order* (2011).

②Broecke, Steven Vanden: *The Limits of Influence: Pico, Louvain, and the Crisis of Renaissance Astrology* (Leiden, 2003), 174-81. I have drawn Largely on Broecke for this part of my discussion. See also Westman, *Copernican Question*, 183-5.

efficient, and final) was held to be formal causation. It was known that planets in conjunction might be separated from one another by vast differences, even though they appeared to be close together against the fixed stars. So, their effect on earth was not due to combined *efficient* causation (like shining two bright torches at the earth instead of one), but was due to the perceived pattern they made in the sky—hence it was *formal* causation.

In his attack on astrology, Pico insisted that there were only two kinds of celestial influence—light, and heat or more specifically *calor caelestis* (celestial heat), the kind of life-giving heat that could be perceived to emanate from the sun, and promoted all life on earth. (14, pp.320-321) Pico was now disregarding the usual emphasis upon formal causation and implying that if the heavenly bodies were to have any effect it must be through efficient causation—by the effect of light or heat. Consequently, Pico could now bring in geometrical assumptions, and argue that the further away a planet was from the earth, the weaker would be its light and heat. Accordingly, all the heavenly bodies are too far away, or too small, to have any effect at all, except for the sun and moon, and indeed the sun and moon were so powerful in their efficient causation, due to their size and proximity, that they would overwhelm whatever tiny efficient causes the planets might have [①] . As Broecke has pointed out, “This critique eliminated the complexity of astrological practice by reducing it to two bodies (Sun and Moon) only.” [②] But it seems that it also inspired astrologers at Louvain and in Wittemberg to counter-attack by accepting the *efficient* causation of astrological influence, and using the newly possible (Copernican) ability to calculate planetary distances to try to develop the details. (14, pp.178-83)

Dee was at the forefront of these efforts during his time in Louvain, and his *Propaedeumata aphoristica* is his own attempt to develop a new astrology in which the efficient causes of the heavenly bodies can be accurately assessed by using geometry to take into account the size and distances of the heavenly bodies and the angle of incidence their rays make with the earth. (10, pp.51-73) In Aphorism 31, for example, he advises his readers that “The true distances of the fixed stars and of each of the planets from the center of the earth at any given time should be determined by the astrologer” [③] . After turning his attention to the angle of incidence of heavenly rays, in Aphorism 33 he explains that a ray of influence from a star to an external point is reinforced by a surrounding cone of influence; “The axis of the cone is the ray, the vertex is the external point”; while the base of the cone “is that luminous portion of the convex surface of the same star” which faces the external point. Here he is dealing with surface effects–rays emanating from the surface of a star or planet–but he tells his readers that “we shall speak in another place of rays coming from the depths of stellar bodies”. Meanwhile, he points out that “All stars larger than the earth imprint their rays upon it more strongly in the degree in which they are nearer to it; and they also illuminate a larger portion of the earth by their sensible and direct rays than when they are separated from it by a greater interval.”(10, pp.137-39)

Dee was adopting the principles of geometry, and of geometrical optics, to develop a new account of astrological influence here. And this was intended to refute Pico’s claims that only the sun and moon can exert a sensible effect on the earth, and that the only means by which they can affect the earth is through light and celestial heat. On this last point, Dee implied throughout that there are other kinds of influence. In Aphorism 25, for example, he tells us that “the rays of all stars are double: some are sensible or luminous, others are of more secret influence”. (10, p.133) Later (Aphorism 45), he says that the geometry of influence he is developing “skilled persons perceive to be of

①Dee dismisses Pico’s suggestion that the sun would overwhelm any effects the planets might have in Aphorism 90 of Dee, *Propaedeumata*, 177. But see also, Aphorism 95 (179), where he acknowledges “As the sun surpasses other celestial bodies in size, so it is—one might say—a perpetual and immense source of heavenly light and the chief producer of sensible and vital heat for us.”

②Broecke, *Limits*, 71-2.

③Dee, *Propaedeumata*, 137. Dee clearly believes that it ought to be possible to determine planetary distances, as shown by Copernicus, even though he himself does not accept the truth of Copernican theory. See Broecke, *Limits*, 179-80.

great importance in optics, in astrology, and in magic." (10, p.145) Similarly, writing later in the "Praeface", Dee showed that he did not confine geometrical "perspective" merely to light:

> Perspectiue, is an Art Mathematicall, which demonstrateth the maner, and properties, of all Radiations Direct, Broken, and Reflected. This Description, or Notation, is brief: but it reacheth so farre, as the world is wyde. It concerneth all Creatures, all Actions, and passions, by Emanation of beames perfourmed. Beames, or naturall lines, (here) I meane, not of light onely, or of colour (though they, to eye, giue shew, witnes, and profe, wherby to ground the Arte vpon) but also of other Formes, both Substantiall, and Accidentall, the certaine and determined actiue Radiall emanations[①].

In the "Praeface", eventually, Dee referred back to *Propaedeumata* with some pride, presenting himself as the first to have properly developed this new mathematical method of treating heavenly virtues in such a way: "And in my *Propædeumes* (besides other matter there disclosed) I have Mathematically furnished up the whole Method: To this our age, not so carefully handled by any, that ever I saw, or heard of." [②]

IV. Light Metaphysics and Dee's New Discipline of "Astrological Physics"

Going beyond Dee's mixed mathematical methodology, furthermore, it also worth trying to understand just how the models provided by geometrical optics actually applied to, and explained the behaviour of, the phenomena of the real world. Dee's mathematical method applied to natural philosophy so well precisely because he developed an "astrological physics". (12, pp.42-52) For Dee astrology was "an Arte Mathematicall, which reasonably demonstrateth the operations and effects, of the natural beames, of light, and secret influence: of the Sterres and Planets: in every element and elementall body..."[③]

Dee's physics was astrological in the sense that astrology was held to provide the model for all causation and all changes in the world. Light was held to be the fundamental causal principle in the natural world, and to provide the model for all other efficient causes supposed to be operating in nature. All change was brought about by the action of light, or by rays of some other influence emanating from an active source (and which also operated in accordance with the principles of geometrical optics). As we have just seen, Dee was not content to describe the nature of celestial and other occult virtues merely in qualitative terms. He tried also to calculate the strength of these rays, and the behaviour of the species of lights, in geometrical terms. In so doing he promoted the new claims of mathematicians that mathematics, contrary to what the scholastics claimed, really was able to provide causal explanations in natural philosophy. The geometry of rays provided the key to understanding all natural phenomena; all influences, forces, and powers were propagated in accordance with the principles of geometrical optics. (12, p.54; 162) What Dee's *Propaedeumata* offered was in fact a natural philosophy based on radiated influences, and these influences were held to be the principal causal actions operating throughout the cosmos. (12, p.169; 233) The "outstanding virtues of nature", mentioned on the title page, were radiated from all things. As Dee wrote in Aphorism 4: "Whatever exists by action emits spherically upon the various parts of the universe rays which, in their own manner, fill the whole universe. Wherefore every place in the universe contains rays of all the things that have active existence." (10, p.123) Furthermore, in Aphorism 27 we read that celestial rays have the power and virtue of penetrating everything in the universe, and that this proves the stars "have a great readiness to influence everything", or to impart their "virtue" to everything. (12, p.135)

The medieval English philosopher, Roger Bacon

①Dee, Mathematical Pracface, sig. b.j.r.

②Dee, Mathematical Pracface, sig. b.iiiv-b.iiijr.

③Dee, Mathematical Pracface, sig. b.iiir.

and the tradition of which he was a part, known to historians as “light metaphysics” might have served as a source for Dee here ①. The so-called “light metaphysics” tradition is usually seen as deriving from the Emanationism of the Neo-Platonist philosopher Plotinus, and developed by Proclus, Pseudo-Dionysius, and others, in which the world system was seen as consisting of a series of emanations from God, beginning with the most spiritual (light), and ending with the densest matter. The ancient neo-Platonic emanationists considered light to be the efficient cause by which God brought about the Creation. Their speculations were not restricted to visible light, because they also believed that light could act as a hidden cause of phenomena, and even a hidden source of force as though light could push things as it disseminated from a source. Accordingly, the tradition provided an alternative way of understanding physical causation, and this alternative kind of explanation was prominent in the magical tradition. This can be seen most clearly in the medieval Latin work known as *De radiis stellarum* (*On the rays of the stars*), or even as *De theorica artium magicarum* (*On the theory of the magical arts*), which was attributed to the Arabic philosopher, al-Kindi (although no Arabic version of the work exists) ②. As al-Kindi writes at one point: “it is obvious that each thing in this world, whether substance or accident, produces rays in its own way like the stars”. And “Assuming this to be true we say that everything which actually exists in the world of elements sends out rays in all directions. These [rays] fill the entire world of elements in their own way.” [15]

The tradition was introduced into England in the thirteenth century by Robert Grosseteste, Bishop of Lincoln, who translated the works of Pseudo-Dionysius from Greek into Latin, and circulated them with his own commentary. [16, 17] Grosseteste clearly believed that light did much more than illuminate the world; he also held it to be a universal principle of activity and a major cause of change in the universe. As he wrote, “the first corporeal form, which some call corporeity I consider to be light. For light by itself diffuses itself in every part, in such a way that a point of light, a sphere of light of any size (as big as you like) is generated”. He went on to say that matter and form are diffused throughout the universe in the same way, but it is the spontaneous diffusion of light which drives this: “I have proposed that it is light which possesses of its very nature the function of multiplying itself and diffusing itself instantaneously in all directions.” [18] In another work, Grosseteste wrote that “Truly, when light generates itself following one path and dragging matter with itself, it creates local motion. Indeed, when light, which is inside matter, is sent out and what is outside sent in, it creates change. And it is clear, that the motion of bodies is [the result of] a multiplicative power of light③.

These ideas, and other aspects of the tradition, were taken up by Roger Bacon. Bacon was so committed to light metaphysics that he incorporated it into his entire system of *physics*. But he never developed a metaphysics of light. Rather, Bacon suggested the physics of light as al-Kindi and Grosseteste had claimed, in which doctrine the “radiation is the universal instrument of natural causation—and developed it into a systematic doctrine.” [19] In his important work *On the Multiplication of Species* (*De multiplicatione specierum*), Bacon extended the meaning of “species” beyond its original meaning of “likeness” or “image”,

①For Bacon’s influence on Dee, see Clulee, *John Dee’s Natural Philosophy* (2014), 52-73. On Bacon and the tradition of light metaphysics, see Lindberg, *Theories of Vision from Al-Kindi to Kepler* (1976); Bacon, *Roger Bacon’s Philosophy of Nature: A Critical Edition*, with English Translation, Introduction, and Notes, of De Multiplicatione Specierum and De Speculis Comburentibus, ed. and trans. Lindberg (1983). On “light metaphysics”, see McEvoy, *Robert Grosseteste* (Oxford, 2000), 90-5, where he argues that there were metaphysical aspects of the tradition.

②On this work, and its authorship, see Adamson, *Al-Kindī* (2007), 188-206; Al-Kindī, *The Philosophical Works of Al-Kindī*, ed. Adamson and Pormann, trans. Adamson and Pormann (2012), 218-19.

③This is from Grosseteste’s De motu corporali et luce, 51-2, my translation: “Cum vero lux secundum unam viam se generat secum trahens materiam, fit motus localis. Cum vero lux, quae est intra materiam, mittatur foras et quod foris est, immittit intus, fit alteratio. Et in hoc patet, quod motio corporalis est vis multiplicativa lucis.” See also, Crombie, *Robert Grosseteste and the Origins of Experimental Science*, 1100-1700, 107.

so that, they became the powers or virtues by which objects acted on their surroundings, "a synonym for al-Kindi's universal force." As Bacon wrote, the "species" is the first effect of an agent; in this regard, all the other effects are produced. Thus, those wise and foolish men differ from their knowledge concerning species. Nevertheless, they agree in this, that the agent sends species into the matter on which it acts, so that, through that which was first produced, it can bring out of the potentiality of the matter the complete effect the agent intended[①].

Dee was a great admirer of Bacon, and it could be said that the system of natural philosophy presented in the *Propaedeumata* owes more to Bacon than to any other earlier thinker, although there are clear similarities also with al-Kindi, and Grosseteste[②]. Dee accepted the standard view of the so-called light metaphysics tradition that light was not always visible, but sometimes operated in a hidden way. Moreover, the "secret" influences were more pervasive. In Aphorism 25, for example, we read: "The rays of all stars are double, some sensible or luminous, others of more secret influence. The latter penetrate in an instant of time everything that is contained in the universe; the former can be prevented by some means from penetrating so far. (10, p.133) The opening aphorisms of the *Propaedeumata* make it clear that all change, including the motions of things, are the result of the rays which all things emit. So, in Aphorism 6 we are told: "Just as one thing differs from another, so their rays differ in their power of affecting and in the causing of their effects so long as they act wholly upon the same object." And in Aphorism 14, we read: Not merely spiritual species, but also other natural ones, flow from things both through light and without light, not to sight only but sometimes to other senses; and they come together especially in our imaginal spirit, as if in a mirror, show themselves to us, and enact wonders in us." (10, p.125; pp.127-29)

At one important point, Dee argued that a knowledge of catoptrics, the geometrical optics of plane and curved mirrors, could enable us to make the operation of occult virtues manifest: "If you were skilled in 'catoptrics', you would be able, by art, to imprint the rays of any star much more strongly upon any matter subjected to it than nature itself does." Dee suggested that the use of curved mirrors could intensify the effects of occult celestial influences, just as they can be used to intensify the sun's heat. Accordingly, we could manipulate the rays making them either stronger or weaker, and thus could manipulate the influence of those rays upon the terrestrial realm. As Dee added in a corollary to this aphorism (my emphasis):

> By this means obscure, weak, and, as it were, *hidden virtues of things*, when strengthened *by the catoptric art, may become quite manifest to our senses*. The industrious investigator of secrets has great help offered to him from this source in testing the peculiar powers not merely of stars but also of other things which they work upon through their sensible rays.

Dee also suggests in this aphorism that he is merely reviving the natural magic of "the ancient wise men", but he also links it to the manipulations of nature performed by alchemists, or philosophers of so-called "inferior astronomy": "This, indeed, was by far the largest part of the natural magic［*naturalis Magiae*］of the ancient wise men. And this secret is not of much less dignity than the very august astronomy of the philosophers, called inferior…"(10, p.149) Indeed, Dee's mathematical astrological physics depended upon the same combination of geometrical optics with natural magic about celestial rays that can be found in al-Kindi's *De theorica artium magicarum*, Grossestete's *De luce*, and especially in Roger Bacon's *De multiplicatione specierum* and his *De speculis comburentibus* (*On burning mirrors*).

Dee's combining of these traditions was partly shared with his contemporaries, particularly those

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① Bacon, De multiplicatione specierum, 1.1. 42-69, 75-80; "scilicet quod species est primus effectus agentis. Per hanc enim omnes aestimant effectus caeteros produci, unde sapientes et insipientes circa multa in specierum cognitione differunt. Communicant tamen in hoc, quod agens influit speciem et materiam patientis, quatenus per eam primo factam possit educere de potentia materiae effectum completum quern intendit".

② Dee seems to have been introduced to the work of Roger Bacon while he was in Louvain, and subsequently collected a number of Bacon manuscripts. See Clulee, *John Dee's Natural Philosophy*, 12-3, 28.

who, like Dee, believed that natural magic and mathematical skills may be more potentially fruitful than scholastic natural philosophy in search of “the secret virtues of nature”. For Thomas Digges at Cambridge, for example, Dee may well have offered a natural philosophy that was supported by, and perhaps even proved true by, the certainties of mathematics. Thomas Digges, the son of Leonard Digges, who had published a few practical mathematical treatises, referred to Dee as his “second parent” in mathematics; and Dee returned the compliment by calling Digges his “heir.” Digges made his acceptance of Copernicanism clear in “A Perfit Description of the Caelestiall Orbes”, which he added to his edition of his father’s *Generall Prognostication* in 1576, along with an English translation of four chapters from Copernicus’s *De revolutionibus*. [20, 21] Digges’s Copernicanism already strongly suggests that he believed mathematics can reveal truths about the nature of the world, but we can see this confirmed in the preface to his “Perfit Description” [①] . Although he did not go into details Digges suggested that the sun drives the planets, including the earth, around itself because it “giveth laws of motion to the rest”; this is a philosophical claim. A few lines later Copernicus was presented as a philosopher who proves the truth of his claims by mathematics: “reason and deepe discourse of wit having opened these things to Copernicus, and the same being with demonstrations Mathematicall, most apparently to the world delivered.” (20, sig. Mr) Over the page he dismissed those who regard Copernican theory as merely an exercise in hypothetical mathematics: “Copernicus ment not as some have fondly excused him, to deliver these grounds of the earth’s mobility, onely as Mathematicall principles fayned, and not as Philosophicall truly averred: I have also from him delivered both the Philosophical reasons by Aristotle and others, produced to maintaine the Earths stabilitie, and also their solutions and insufficiencie… [②] Digges did not completely forget the mathematical side of the matter. A little later he asked his readers: “If therefore the Earth be situate immoveable in the Center of the world, why finde we not Theoricks upon that ground to produce effects as true and certain as those of Copernicus? Why cast wee not away those *Circles Aequantes*, and motions irregular?

He went on to suggest that if it proves impossible to provide mathematical theories which explain the earth’s stability, then we should allow ourselves to be ruled instead by the “Rule of Reason”. (20, sig. Mv.)

Conclusion

In this article I have located Dee’s early works, the *Propaedeumata aphoristica* and the “Mathematicall Praeface”, within the contexts of early modern disciplinary complexities. I have established that Dee, as one of the most significant natural philosophers in the Renaissance, promoted the idea, opposed to scholastic assumptions, that mathematics should be used to help us to understand the workings of the physical world. Beyond these general terms, I have also argued that Dee demonstrated the close relevance of mathematics to natural philosophy in detail by means of the mixed mathematical methods of the geometrical optics. Dee offered a new discipline of astrological physics based on his belief that rays of force, emanated by all bodies, and operating throughout the world, operated like rays of light. This new concept of physics was derived from Dee’s belief in celestial influence and his notion that such influences emanated from heavenly bodies just as light did—a notion derived ultimately from light metaphysics traditions of the Middle Ages. Accordingly, geometrical optics could be extended to reveal not just the behaviour of light, but also the behaviour of occult virtues and influences. This mixed mathematical method, therefore, became Dee’s way of dealing with the scholastic notion of occult virtues; the disciplinary changes which promoted the use of mathematics in natural philosophy also became the means of justifying belief in the reality of occult influences. “If you were skilled in ‘catoptrics’”, as Dee

①Copernicus himself took this line, of course. See Westman, “Astronomer’s Role.”

②Digges discusses Aristotle’s arguments against a moving earth sigs N2r-N3r, and “The solution of these Reasons with their insufficiencie”, sigs N3r-Or.

claimed, “hidden virtues of things, when strengthened by the catoptric art, may become quite manifest to our senses”.

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